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The Ring Theory in the Zoning of Images: An Application

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ABSTRACT

Ring theory is one of the branches of the abstract algebra that has been broadly used in images. However, ring theory has not been very related with image segmentation. In this paper, we propose a new index of similarity among images using $\mathbb{Z}_n$ rings and the entropy function. This new index was applied as a new stopping criterion to the Mean Shift Iterative Algorithm with the goal to reach a better segmentation. An analysis on the performance of the algorithm with this new stopping criterion is carried out. The obtained results proved that the new index is a suitable tool to compare images.

KEYWORDS: *Image processing, similarity index, entropy, ring theory*

I. INTRODUCTION

Many techniques and algorithms have been proposed for digital image segmentation. Traditional segmentation such as thresholding, histograms or other conventional operations are rigid methods. Automation of these classical approximations is difficult due to the complexity in shape and variability within each individual object in the image. The mean shift is a non-parametric procedure that has demonstrated to be an extremely versatile tool for feature analysis. It can provide reliable solutions for many computer vision tasks [2]. Mean shift method was proposed in 1975 by Fukunaga and Hostetler [5]. It was largely forgotten until Cheng's paper retook interest on it [1]. Segmentation by means of the Mean Shift Method carries out as a first step a smoothing filter before segmentation is performed [2, 3].

Entropy is an essential function in information theory and this has had a special uses for images data, e.g., restoring images, detecting contours, segmenting images and many other applications [12, 15]. However, in the field of images the range of properties of this function could be increased if the images are defined in $\mathbb{Z}$ rings. The inclusion of the ring theory to the spatial analysis is achieved considering images as a matrix in which the elements belong to the cyclic ring $\mathbb{Z}_n$. From this point of view, the images present cyclical properties associated to gray level values.

Ring Theory has been well-used in cryptography and many others computer vision tasks [18]. The inclusion of ring theory to the spatial analysis of digital images, it is achieved considering the image like a matrix in which the elements belong to finite cyclic ring $\mathbb{Z}_n$. The ring theory for the Mean Shift Iterative Algorithm was employed by defining images in a ring $\mathbb{Z}_n$. A good performance of this algorithm was achieved. Therefore, the use of the ring theory could be a good structure when one desire to compare images, due to that the digital images present cyclical properties associated with the pixel values. This property will allow to increase or to diminish the difference among pixels values, and will make possible to find the edges in the analyzed images. In this paper, a new similarity index among images is defined, and some interesting properties based on this index are proposed. We compare also the instability of the

iterative mean shift algorithm (MSHi) by using this new stopping criterion with regard to the stopping criterion used in [8-11]. Furthermore, we make an extension of [7], and we expand the theoretical aspects by studying in depth the cyclical properties of rings applied to images. For this purpose, and in order to mark the difference of this paper with regard to [7], some issues are pointed out below:

II. Revision of the Mean Shift Theory

• Important elements of the ring G_{kxm} $(\mathbb{Z}_n)(+,\cdot)$ are given: neutral, unitary, and inverse. In particular, the inverse element was used so much to the theoretical proofs as well as practical aspects.

• Explanation of strong equivalent images by using histograms.

• Definition of equivalence classes.
• Quotient space. Definition and existence.
• Natural Entropy Distance (NED) definition.
• Configuration of the algorithm MSHi with the NED distance.

The remainder of the paper is organized as follows. In Section 2, the more significant theoretical aspects of the mean shift and entropy are given. Section 3 describes the similarity index, its consequences for entropy function and the significance of the cyclic ring $\mathbb{Z}_n$ for images. Also, it is defined the quotient space of strongly equivalent images and some properties of entropy are proved. The experimental results, comparisons and discussion are presented.

III. Natural Entropy Distance and MSHi and Results

Taking in consideration the good properties that, in general, the NED definition has, one sees logical to take this new similarity index as the new stopping criterion of MSHi. Explicitly, the new stopping criterion is:

$$E\big(A_k + (-A_{k-1})\big) \le \epsilon, \qquad (1)$$

where ω and k are respectively the threshold to stop the iterations and the number of iterations. Since MSHi is an iterative algorithm, we obtain a sequence that the processed image becomes more homogenous as the algorithm advances. It is intuitive that comparing a resultant image A_k at a given iteration with the previous image A_{k-1} of the corresponding sequence, give us a measure about how much these images look like each other.

We denote this algorithm as $MSHi_{NED}$, and $MSHI_{WE}$ when the similarity index (12) is used as stopping criterion.

The principal goal of this section is to evaluate the new stopping criterion in the MSHi and to illustrate that, in general, with this new stopping criterion the algorithm has better stability in the segmentation process.

For this aim, we used three different images for the experiments, which have been chosen according to the differences among their respective levels of high and low frequencies. The first image ("Bird") has low frequencies, the second ("Baboon") has high frequencies and in the image ("Montage") has mixture low and high frequencies.

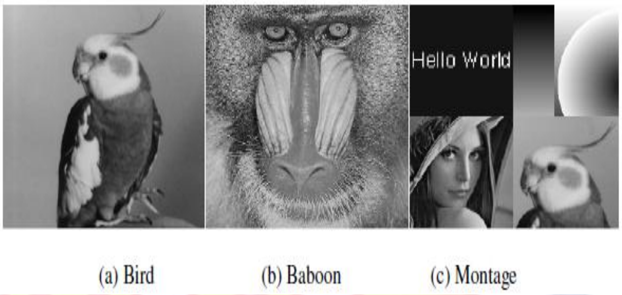

Figure 1. Test images

All segmentation experiments were carried out by using a uniform kernel. In order to be effective the comparison between the old stopping criterion and the new stopping criterion, we use the same value of h_r and h_s in MSHi (h_r= 12, h_s= 15). The value of h_s is related to the spatial resolution of the analysis, while the value h_r defines the range resolution.

In the case of the new stopping criterion, we use the stopping threshold ϵ = 0.9 and when the old stopping criterion was used ϵ = 0.01. Figure 4 shows the segmentation of the three images. Observe that, in all cases, the MSHi had better result when the new stopping criterion was used.

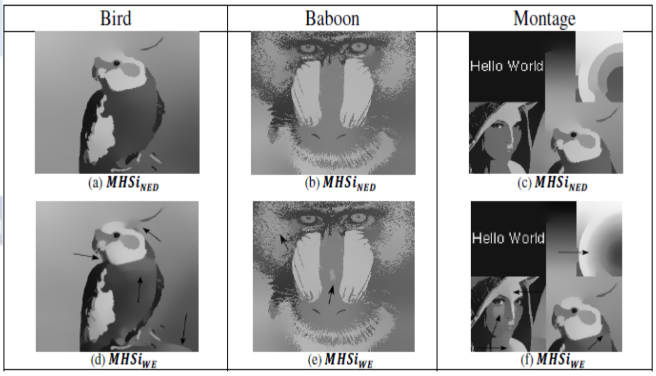

Figure 2. Segmentation of the experimental images. In the first row are shown the segmentation using the new stopping criterion and in the second row are the segmented images using the old stopping criterion.

When one compares Figures 4(a) and 4(d), in the part corresponding to the face or breast of the bird,

more homogeneous area, with the new stopping criterion was obtained (see arrows in Figure 4(d)). Observe that, with the old stopping criterion the segmentation gives regions where different gray levels are originated. However, these regions really should have only one gray level. For example, Figure 4(b) and 4(e) show that the segmentation is more homogeneous when the new stopping criterion was used (see the arrows). In the case of the "Montage" image one can see that, in Figure 4(f) exist many regions that contain different gray levels when these regions really should have one gray level (see for example the face of Lenna, the circles and the breast of the results are obtained because the defined new stopping criterion through the natural distance among images in expression (14) offers greater stability to the MSHi of the obtained segmented images by using the two stopping The plates that appear in Figure 5(b) and 5(d) are indicative of equal intensity levels. In both intensity to other represent the different regions in the segmented Note that, in Figure 5(b) exists, in the same region of the segmentation, least variation of the pixel intensities with regard to Figure 5(d). This illustrates that, in this case the segmentation was better when the new stopping criterion was used.

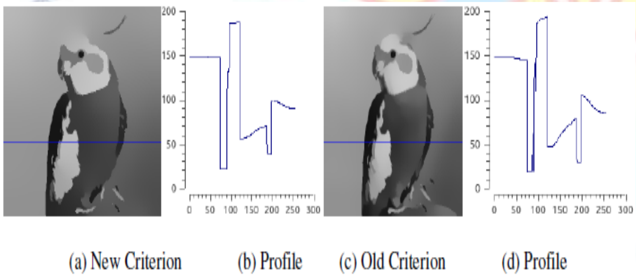

Figure 3. An intensity profile through of a segmented image.

Profile is indicated by a line. (a) and (c) are the segmented images and (b) and (d) are the profile of (a) and (c) respectively. Figure 5 shows the performance of the two stopping criterion in the experimental images. "y" axis appears the iterations of stopping criterion in each iteration of the algorithm.

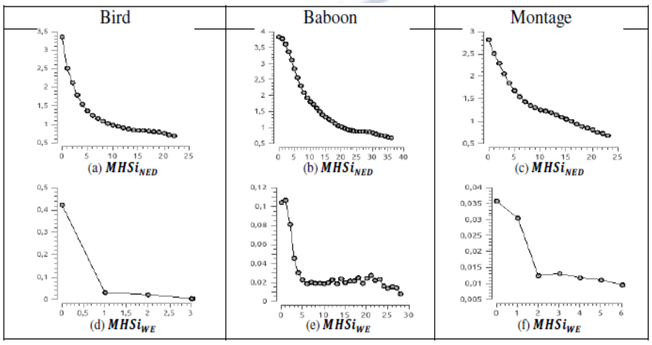

Fig 4: Behavior of the stability of each algorithm according to the test images.

The graphics of iterations of the new stopping criterion show a smoother behavior; that is, the new stopping criterion has a stable performance through the MSHi. The new stopping criterion not only has good theoretical properties, but also, in the practice, has very good behavior. On the other hand, if we analyze the old stopping criterion in the experimental images, one can see that the performance in the MSHi is unstable. In general, we have this type of situation when the stopping criterion defined in (12) is used. This can originate bad segmented images.

IV. Conclusions

In this work, a new stopping criterion, for the iterative MSHi, based on the Ring Theory was proposed. The new stopping criterion establishes a new measure for the comparison of two images based on the use of the entropy concept and the spatial information. The quotient space was defined using the equivalent classes of images, to be able of selecting any element of the class. Through the obtained theoretical and practical results, it was possible to prove that the new stopping criterion had very good performance in the algorithm MSHi, and was more stable that the old criterion.

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